

The neutron matter equation of state from low-momentum interactions

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Abstract. We calculate the neutron matter equation of state at finite temperature based on low-momentum nucleon-nucleon and three-nucleon interactions. Our results are compared to the model-independent virial equation of state and to variational calculations. We provide a simple estimate for the theoretical error, important for extrapolations to astrophysical conditions.

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The nuclear equation of state plays a central role in the physics of neutron stars [1] and core-collapse supernovae [2, 3]. Renormalization group methods coupled with effective field theory offer the possibility of a new and systematic approach to nuclear matter: For low-momentum interactions $V_{\text{low } k}$ [4] with cutoffs around 2 fm^{-1} , the strong short-range repulsion in conventional nucleon-nucleon (NN) interactions and the tensor force are tamed [5, 6]. At sufficient density, Pauli blocking eliminates the shallow bound states, and thus the particle-particle channel becomes perturbative in nuclear matter [5]. In addition, the corresponding leading-order chiral three-nucleon (3N) interaction becomes perturbative in light nuclei for $\Lambda \lesssim 2 \text{ fm}^{-1}$ [7]. Consequently, the Hartree-Fock (HF) approximation is a good starting point, and perturbation theory (in the sense of a loop expansion) around the HF energy becomes tractable [5]. The perturbative character is due to a combination of Pauli blocking and an appreciable effective range (see also [8]).

At finite temperature, the loop expansion around the HF free energy can be realized, based on the work of Kohn, Luttinger and Ward [9, 10], by the perturbative expansion of the free energy, where the momentum dependence of the self-energy is treated perturbatively. In this work, we include the first-order NN and 3N contributions, as well as anomalous and normal second-order diagrams with NN interactions. The pressure, entropy and energy are calculated using standard thermodynamic relations. We use the cutoff dependence to provide simple error estimates, and find that the cutoff dependence is reduced significantly, when second-order contributions are included.

We start from the perturbative expansion of the grand-canonical potential $\Omega(\mu, T, V)$, where μ is the chemical potential, T the temperature and V the volume:

$$\Omega = \Omega_0 + \Omega_{1,\text{NN}} + \Omega_{1,3\text{N}} + \Omega_{2,\text{a}} + \Omega_{2,\text{n}} + \dots \quad (1)$$

The non-interacting system is given by Ω_0 , $\Omega_1 = \Omega_{1,\text{NN}} + \Omega_{1,3\text{N}}$ denotes the first-order NN and 3N, and $\Omega_{2,\text{a}} + \Omega_{2,\text{n}}$ are the second-order anomalous and normal contributions.

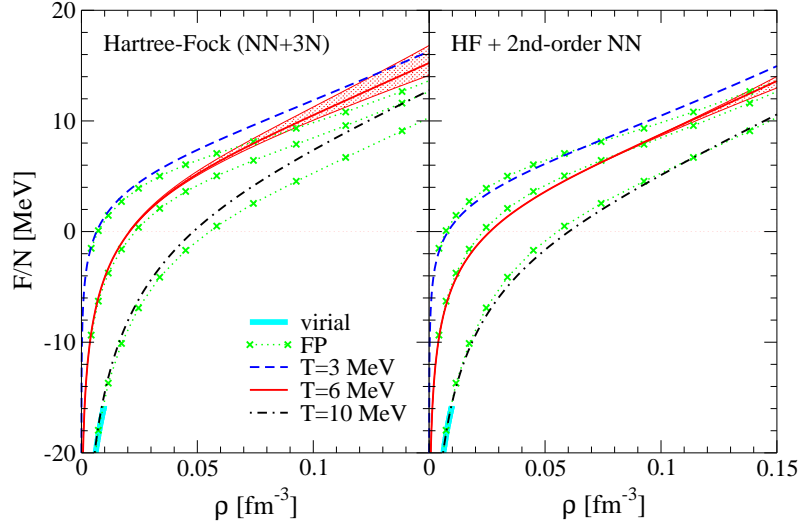


FIGURE 1. The free energy per particle F/N as a function of density ρ . The left figure gives the first-order NN and 3N contributions with a free single-particle spectrum. Second-order anomalous and normal NN contributions are included in the right figure. Our results are compared to the virial equation of state (virial) [12] and to the variational calculations of Friedman and Pandharipande (FP) [13]. The virial curve ends where the fugacity $z = e^{\mu/T}$ is 0.5.

The free energy $F(N, T, V)$ is obtained by a Legendre transformation of the grand-canonical potential with respect to the chemical potential, $F(N, T, V) = \Omega(\mu, T, V) + \mu N$, with mean particle number N . Following Kohn and Luttinger [9], we have

$$F(N) = F_0(N) + \Omega_1(\mu_0) + \Omega_{2,n}(\mu_0) + \left[\Omega_{2,a}(\mu_0) - \frac{1}{2} \frac{(\partial \Omega_{1,NN}/\partial \mu)^2}{\partial^2 \Omega_0/\partial \mu^2} \Big|_{\mu_0} \right] + \dots, \quad (2)$$

where μ_0 is the chemical potential of a non-interacting system with the same density $\rho = N/V$ as the interacting system, $N = -[\partial \Omega_0/\partial \mu]_{\mu_0}$, and $F_0(N) = \Omega_0(\mu_0) + \mu_0 N$ is the free energy of the non-interacting system. The above expansion ensures that the $T \rightarrow 0$ limit is correctly reproduced [9, 10]. The anomalous second-order diagram accounts for perturbative corrections to the free single-particle spectrum.

Our results [11] for the free energy per particle are shown in Fig. 1 for temperatures $T = 3 \text{ MeV}, 6 \text{ MeV}$ and 10 MeV , where the low-momentum interaction $V_{\text{low}k}$ is obtained from the Argonne v_{18} potential for a cutoff $\Lambda = 2.1 \text{ fm}^{-1}$. For the 3N contribution at the HF level, we find that only the c_1 and c_3 terms of the long-range 2π -exchange part survive (for details on the 3N interaction, see [5, 7]). For the $T = 6 \text{ MeV}$ results, we provide error estimates by varying the cutoff over the range $\Lambda = 1.9 \text{ fm}^{-1}$ (lower curve) to $\Lambda = 2.5 \text{ fm}^{-1}$ (upper curve). As expected the error grows with increasing density. From Fig. 1, we observe that the equation of state becomes significantly less cutoff dependent with the inclusion of the second-order NN contributions.

In Fig. 1, we also compare our results for the free energy to the model-independent virial equation of state [12] and to the variational calculations of Friedman and Pandharipande [13] (FP, based on the Argonne v_{14} and a 3N potential). We find a very good agreement with the virial free energy, and for the densities in Fig. 1 similar results as FP.

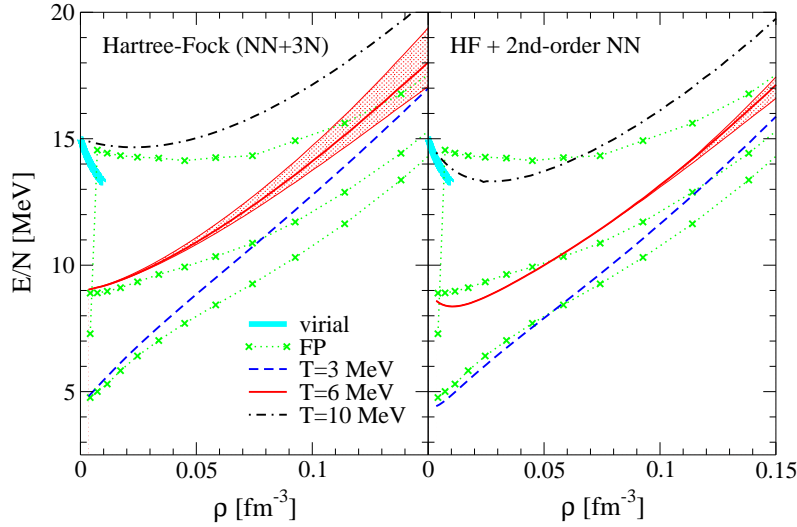


FIGURE 2. The energy per particle E/N as a function of density ρ to first and second order as in Fig. 1.

Our results [11] for the energy per particle are presented in Fig. 2. As for the free energy, we observe additional binding and a significantly reduced cutoff dependence at second order. In contrast to the variational calculation of FP [13], the low-density behavior at second order is in good agreement with the virial equation of state [12]. This highlights the importance of a correct finite-temperature treatment of second and higher-order contributions. This work is part of a program to improve the nuclear equation of state input for astrophysics, and to provide error estimates, for example, for the neutron star mass and radius predictions.

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